Statistics:
Expected Value and Variance

When we have collected a large dataset we would like to look at descriptive quantities of the data (e.g. mean) rather than the entire dataset itself. The same is through for probability distributions of random variables.

Expected Value

For a discrete random variable (a random variable take can take only discrete value e.g. dice) the expected value is

$$\mu = E(X) = \sum_x xP(X = x)$$

For a continuous random variable the expected value is

$$E(X) = \int_x xf(x)dx$$

where $f(x)$ is the probability density function of x. The probability density function is a function describing the distribution of the random variable x.

Variance

For a discrete random variable the variance is

$$\begin{aligned} Var(X) &= E((X - E(X)^2) \\ &= E(X^2) - E^2(X) \\ &= \sum_x x^2 P(X = x) - E^2(X) \end{aligned}$$

and similarly for a continuous random variable

$$Var(X) = \int_x x^2 f(x)dx - E^2(X)$$

Example

Imagine you play a game with a dice, where each time you win the amount shown on the dice after you roll. You could win only €1 but you could also win as much as €6. What would you expected to win though and what would be the variance of this amount?

Let X be the amount you win, $X \in (1, 2, 3, 4, 5, 6)$. Then $E(X)$ is

$$1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = \frac{21}{6} = 3.5$$

and $\mathrm{Var}(X)$ is

$$1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} - (3.5)^2 = \frac{91}{6} - (3.5)^2 = \frac{35}{12} = 2.9167$$